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MECHANICAL SWEEP METHOD FOR CALCULATING THE NON-LINEAR DEFORMATION OF A CYLINDRICAL PANEL†

V. V. VASIL'YEV

Ufa

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A mechanical sweep method is formulated. Using the calculation of the non-linear deformation of a cylindrical panel as an example, an algorithm is developed that transfers the boundary conditions expressed in terms of mechanical parameters over the elements into which the panel is decomposed. The validity of the algorithm is confirmed by the agreement between numerical results and experimental data.

Sweep methods are based on transferring the boundary conditions in the numerical solution of differential equations [1, 2]. Compared to other numerical methods, such as finite-element methods, the advantages of the sweep method are the specification of a smaller number of unknowns (only on the boundary), and simpler algorithmization. However, in previously-known versions of the sweep method the differential equations are solved numerically separately from the mechanical parameters of the problem, and one loses a number of ways of further improving the method.

When calculating the stress–strain state of shell structures the sweep method is easily implemented using the strain consistency condition in the form of cross-over couplings [3].

Sweeping using mechanical parameters has advantages when solving non-linear structural deformation problems. Taking into account the change in the direction of the force factors under strain, one can determine the non-linear dependence of the load on the displacements.

The purpose of this paper is to develop a mechanical sweep method in which the transfer of the boundary conditions from one element of the structure to another is performed directly in terms of mechanical parameters: the forces, couples, angles of rotation and displacements.

1. STATEMENT OF THE PROBLEM

We will consider the sweep method using the example of the two-dimensional case of large displacements of a rigidly clamped cylindrical square panel. The panel is of width b, radius R and thickness h, and is loaded with a pressure q (Fig. 1). We introduce a system of cylindrical coordinates $O_1l_1\rho\varphi$. We mentally divide the panel by lines along n intervals of generators of the cylinder parallel to l_1 and perpendicular to it. The subscript i corresponds to sections parallel to the cylinder generator l_1, and the subscript j to sections perpendicular to the l_1 axis. From the n^2 square elements of width a we select the one with the jth lowest and ith leftmost section (Fig. 2).

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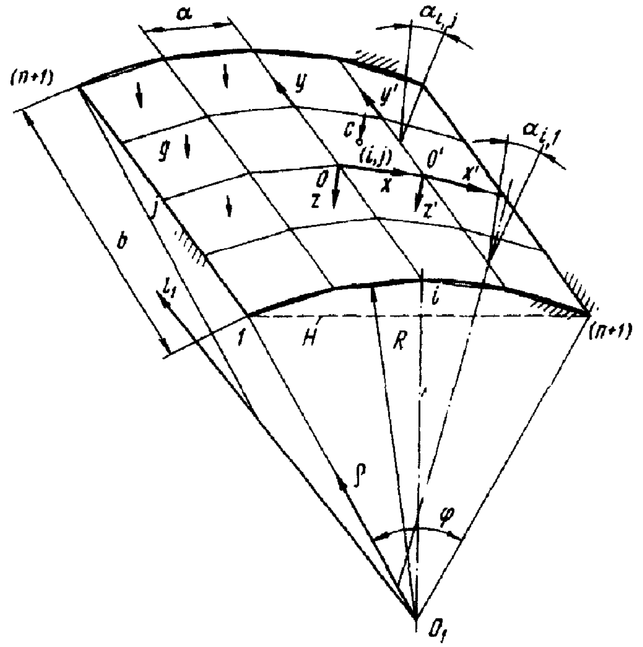
α
$\alpha_{i,j}$
(n+1)
y
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b
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i
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H
R
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ρ
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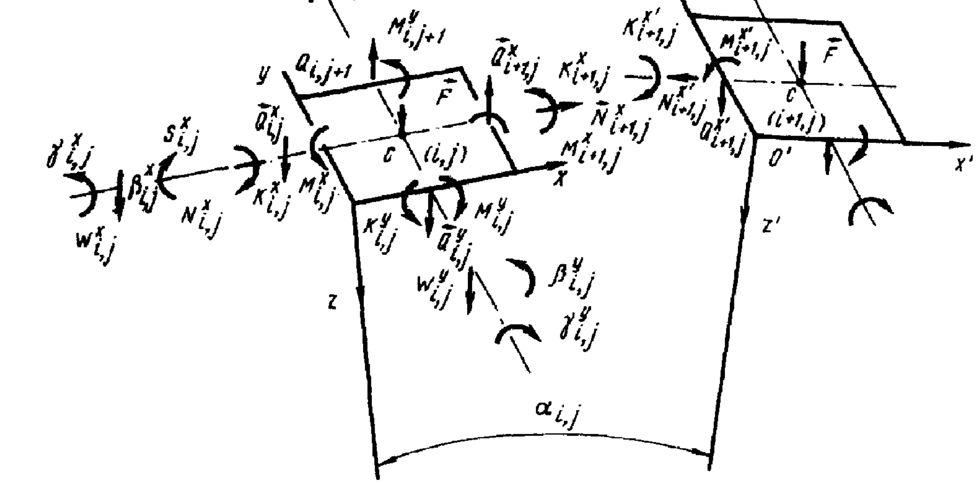
Fig. 1.


F
C
(i,j+1)
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$M^y_{i,j+1}$
$Q_{i,j+1}$
$Q^x_{i+1,j}$
$K^x_{i+1,j}$
$K^{x'}_{i+1,j}$
$M^{x'}_{i+1,j}$
$N^{x'}_{i+1,j}$
$N^x_{i+1,j}$
$M^x_{i+1,j}$
$Q^{x'}_{i+1,j}$
(i+1,j)
0'
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y'
z'
y
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(i,j)
$\gamma^x_{i,j}$
$S^x_{i,j}$
$Q^x_{i,j}$
$\beta^x_{i,j}$
$N^x_{i,j}$
$K^x_{i,j}$
$M^x_{i,j}$
$W^x_{i,j}$
$K^y_{i,j}$
$Q^y_{i,j}$
$M^y_{i,j}$
$W^y_{i,j}$
$\beta^y_{i,j}$
$\gamma^y_{i,j}$
$\alpha_{i,j}$

Fig. 2.

In our algorithm we take the number of elements in the decomposition to be large. This enables us to assume the elements to be plane with a sufficient degree of accuracy. At each element we introduce a local system of coordinates $Oxyz$.

A variable stress distribution acts over each section of the panel element. According to the principles of rigid body mechanics their stressed state can be represented by forces concentrated at the centre, and couples. The maximum number of force factors for a section element is six. As the element size decreases strain state can be, respectively, represented by six strain parameters at the centre. Because of the limited length of this paper we shall describe only the essence of the proposed method for the case of quadrilateral elements in which the sections parallel to the cylinder generators have four degrees of freedom, while the sections perpendicular to the generators have three degrees of freedom.

In the given case of symmetric panel loading, for sections parallel to the cylinder generators (parallel to the local Oy axis) we shall take into account the bending moment M^x about the middle line, a twisting moment K^x, a shearing force Q^x, and a normal force N^x. We denote the angle of rotation in the M^x direction by β^x, the angle of rotation from the twisting moment by γ^x, the displacement in the direction of Q^x by W^x, and the displacement of the section in the direction N^x by S^x, respectively.

The positive directions of the deformations are shown in Fig. 2. In sections parallel to the Ox axis we denote the bending moment by M^y, the twisting moment by K^y, the shearing force by Q^y, and the corresponding displacements by β^y, γ^y and W^y. These force factors and deformations will be indexed with their corresponding section numbers.

The distributed load q for the element is replaced by a force F concentrated at the centre of the element with

$$F = qa^2 \tag{1.1}$$

2. FUNDAMENTAL EQUATIONS

To construct an algorithm for the mechanical sweep method we shall use the following equations for the selected element.

The equations of static equilibrium

$$\begin{aligned}
&\Sigma M_{kx} = -K^x_{i+1,j} + M^y_{i,j} - M^y_{i,j+1} + K^x_{i,j} + \frac{a}{2}Q^y_{i,j} + \frac{a}{2}Q^y_{i,j+1} = 0 \\
&\Sigma M_{ky} = -\frac{a}{2}F - aQ^x_{i,j} - M^x_{i,j} - \frac{a}{2}Q^y_{i,j} - K^y_{i,j} + K^y_{i,j+1} + \frac{a}{2}Q^y_{i,j+1} + M^x_{i+1,j} = 0 \\
&\Sigma Z_k = F + Q^y_{i,j} - Q^y_{i,j+1} + Q^x_{i,j} - Q^x_{i+1,j} = 0 \\
&\Sigma X_k = -N^x_{i,j} + N^x_{i+1,j} = 0
\end{aligned} \tag{2.1}$$

and physical equations based on Hooke's law and the hypotheses for the straight normals for the element [3]

$$\begin{aligned}
&\frac{d\beta^x}{dx} = \frac{M^x}{EJ_x} - \frac{\mu M^y}{EJ_y}, \quad \frac{d\beta^y}{dy} = \frac{M^y}{EJ_y} - \frac{\mu M^x}{EJ_x} \\
&\frac{d\gamma^x}{dx} = \frac{K^x}{GJ_k}, \quad \frac{d\gamma^y}{dy} = \frac{K^y}{GJ_k}, \quad \frac{dS^x}{dx} = \frac{N^x}{ahE}
\end{aligned} \tag{2.2}$$

Here E is Young's modulus J_x and J_y are moments of inertia for bending ($J_x = J_y = J = ah^3/12$), μ is Poisson's ratio ($\mu = 0.3$), $G = E/[2(1+\mu)]$, J_k is the moment of inertia for twisting ($J_k = \beta_1 ah^3$), and β_1 is a slowly-varying coefficient ($\beta_1 = 0.32$ for $a/h > 10$).

We apply the deformation compatibility equations in the form of the cross-over coupling method [3]. At the centre of the element the displacements (W_c^x, W_c^y) and angles of rotation β_c^x, β_c^y, γ_c^x, γ_c^y are equal if this centre is regarded as the end of a preceding element going along the x axis from one side, and the end of the orthogonal preceding element going along the y axis from the other side (Fig. 2).

Note that the differential relations (2.2) are written for a two-dimensional stressed state of elementary pieces, and not of beams. Hence the beam terminology is a convention.

The angles of rotation and displacements at the centre are found by integrating the elastic median line of the element and using the derivatives in relations (2.2). Here we move separately along the x axis from the left section (Fig. 2) towards the centre on the right, and along the y axis from the lower section upwards towards the centre of the element. For the element numbered (i,j) we obtain

$$\begin{aligned}
&\beta_c^x = \beta_{i,j}^x + \frac{a}{2EJ}\left(M_{i,j}^x + \frac{a}{4}Q_{i,j}^x - \mu M_{i,j}^y\right) = \gamma_c^y = \gamma_{i,j}^y + \frac{aK_{i,j}^y}{2GJ_k} \\
&\beta_c^y = \beta_{i,j}^y + \frac{a}{2EJ}\left(M_{i,j}^y + \frac{a}{4}Q_{i,j}^y - \mu M_{i,j}^x\right) = \gamma_c^x = \gamma_{i,j}^x + \frac{aK_{i,j}^x}{2GJ_k} \\
&W_c^x = W_{i,j}^x + \frac{a}{2}\beta_{i,j}^x + \frac{a^2}{8EJ}\left(M_{i,j}^x + \frac{a}{6}Q_{i,j}^x - \mu M_{i,j}^y\right) = W_c^y = \\
&= W_{i,j}^y + \frac{a}{2}\beta_{i,j}^y + \frac{a^2}{8EJ}\left(M_{i,j}^y + \frac{a}{6}Q_{i,j}^y - \mu M_{i,j}^x\right)
\end{aligned} \tag{2.3}$$

3. THE MECHANICAL SWEEP METHOD ALGORITHM

The conditions at the boundary of the element β_{ij}^x, β_{ij}^y, γ_{ij}^x, W_{ij}^x, W_{ij}^y, S_{ij}^x depend on the parameters of the preceding elements. Sweeping with respect to mechanical parameters begins with the boundary conditions for a rigidly fixed panel, where they are equal to zero.

For example, starting with the lower-left element of the plate, we express $M_{1,1}^y$, $K_{1,1}^y$, $Q_{1,1}^y$ from system (2.3) in terms of $M_{1,1}^x$, $K_{1,1}^x$, $Q_{1,1}^x$, $N_{1,1}^x$ (Fig. 2). Below, the parameters of all elements will express unknown force factors on the left boundary of the panel $M_{1,j}^x$, $K_{1,j}^x$, $Q_{1,j}^x$, $N_{1,j}^x$ and force factors on the upper boundary of the panel $M_{i,n+1}^y$, $K_{i,n+1}^y$, $Q_{i,n+1}^y$. Having expressed the parameters of the last upper element, we pass to the next vertical element strip from left to right.

When considering the subsequent elements the initial conditions at their boundaries are obtained by integrating along the elastic line of the element along the y axis

$$\begin{aligned}
&\beta_{i,j+1}^y = \beta_{i,j}^y + \frac{a}{2EJ}\Big(2M_{i,j}^y + aQ_{i,j}^y + \frac{a}{4}F + \frac{a}{4}Q_{i,j}^x - \frac{a}{4}Q_{i+1,j} - \\
&-\mu M_{i,j}^x - \mu M_{i+1,j}^x + K_{i,j}^x - K_{i+1,j}^x\Big) \\
&\gamma_{i,j+1}^y = \gamma_{i,j}^y + \frac{a}{2GJ_k}\left(2K_{i,j}^y + \frac{a}{2}Q_{i,j}^x + \frac{a}{2}Q_{i+1,j} + M_{i,j}^x - M_{i+1,j}^x\right) \\
&W_{i,j+1}^y = W_{i,j}^y + a\beta_{i,j}^y + \frac{a^2}{2EJ}\Big(M_{i,j}^y + \frac{a}{3}Q_{i,j}^y + \frac{a}{24}F + \frac{a}{24}Q_{i,j}^x - \\
&-\frac{\mu}{2}M_{i,j}^x - \frac{\mu}{2}M_{i+1,j}^x + \frac{1}{4}K_{i,j}^x - \frac{1}{4}K_{i+1,j}^x - \frac{a}{24}Q_{i+1,j}^x\Big)
\end{aligned} \tag{3.1}$$

and along the x axis. (Appropriate relations are obtained from (3.1) by cyclic permutation of the x and y indices.)

Such relations are also obtained by using Mohr integrals.

The central procedure of the sweep is the determination, for an arbitrary element numbered (i,j), of the force factors M^y_{ij+1}, K^y_{ij+1}, Q^y_{ij+1} on the upper boundary (Fig. 2) in terms of the left lateral force factors $M^x_{i,j+1}$, $K^x_{i,j+1}$, $Q^x_{i,j+1}$, $N^x_{i,j+1}$ of the next higher element with number $(i,\ j+1)$. To do this we find from system (2.1) the right lateral force factors $M^x_{i+1,j}$, $K^x_{i+1,j}$, $Q^x_{i+1,j}$ at the element with number (i,j) in terms of the remaining force factors. We substitute them into system (3.1). We then construct the deformation consistency equations (2.2) for the next vertical element with number $(i,\ j+1)$, substituting into them the initial conditions $\beta^y_{i,j+1}$, $\gamma^y_{i,j+1}$, $W^y_{i,j+1}$ from system (3.1) of the preceding element. The solution of this system gives the functions of the parameters $M^y_{i,j+1}$, $K^y_{i,j+1}$, $Q^y_{i,j+1}$.

We will describe the fundamental processes of the mechanical sweep method in matrix form. To this end we introduce the vector of unknown parameters on the left and upper boundaries of the panel, given by the column matrix

$$\mathbf{X} = \| M^x_{1,1}, K^x_{1,1}, Q^x_{1,1}, N^x_{1,1}, \ldots, M^x_{1,n}, K^x_{1,n}, Q^x_{1,n}, N^x_{1,n}, M^y_{1,n+1}, K^y_{1,n+1}, Q^y_{1,n+1}, \ldots$$
$$\ldots, M^y_{n,n+1}, K^y_{n,n+1}, Q^y_{n,n+1}, F \|^T \tag{3.2}$$

The last component of vector (3.2) is given by a known function of the active force F. The algebraic value of the parameter with number k of the $m=7n+1$ parameters (3.2) is determined by the scalar product of matrices of order $m \times m$ for which there is one non-zero unit element on the main diagonal with number k with the column-vector (3.2). For example

$$K^x_{1,1} = \begin{Vmatrix} 0 & 0 & . & . & . \\ 0 & 1 & . & . & . \\ . & . & . & . & . \end{Vmatrix} \mathbf{X} \tag{3.3}$$

Below, the force parameters and deformation parameters of all elements are expressed in terms of the vector (3.2).

We write the force parameters at the upper boundary of the element numbered (i,j) in the form

$$\begin{Vmatrix} M^y_{i,j+1} \\ K^y_{i,j+1} \\ Q^y_{i,j+1} \end{Vmatrix} = \begin{Vmatrix} 0{,}0914 & -0{,}21 & 0{,}2045 \\ -0{,}021 & 0{,}818 & -0{,}0472 \\ 5{,}49a^{-1} & 1{,}17a^{-1} & -2{,}63a^{-1} \end{Vmatrix} \begin{Vmatrix} V_1^{i,j} \\ V_2^{i,j} \\ V_3^{i,j} \end{Vmatrix} \tag{3.4}$$

The intermediate functions $V_1^{i,j}$, $V_2^{i,j}$, $V_3^{i,j}$ are given by the matrix product

$$\| V_1^{i,j}, V_2^{i,j} V_3^{i,j} \|^T = V \| M^y_{i,j}, K^y_{i,j}, Q^y_{i,j}, M^x_{i,j}, K^x_{i,j},$$
$$Q^x_{i,j}, M^x_{i,j+1}, K^x_{i,j+1}, Q^x_{i,j+1}, \beta^y_{i,j}, \gamma^y_{i,j}, W^y_{i,j}, \beta^x_{i,j+1}, \gamma^x_{i,j+1}, W^x_{i,j+1}, F \|^T \tag{3.5}$$

The coefficient matrix has the form $\xi = EJa^{-1}$

$$V = \begin{Vmatrix} -1 & 0{,}3 & -0{,}1a & 0{,}6 & 0{,}3a & 0{,}3 & 0{,}65 & 0 & 0 & -2\xi & 0 & 0 & 0 & 2\xi & 0 & 0{,}15a \\ 0 & -0{,}65 & 0 & 0 & 0 & 1 & 0 & 0{,}25a & 0 & 0 & -2\xi & 0 & 2\xi & 0 & 0 & 0 \\ -5 & 1{,}2 & -0{,}57a & 2{,}4 & 1{,}2a & 1{,}3 & 0 & 0{,}17a & 0 & 12\xi & 0 & -8\xi a^{-1} & 4\xi & 0 & 8\xi a^{-1} & 0{,}6a \end{Vmatrix} \tag{3.6}$$

The force parameters at the lower boundary of the panel, taking into account the zero values of boundary deformations $\beta^y_{i,1}$, $\gamma^y_{i,1}$, $W^y_{i,1}$, are determined by solving system (2.3)

$$\left\| \begin{matrix} M^y_{i,1} \\ K^y_{i,1} \\ Q^y_{i,1} \end{matrix} \right\| = \left\| \begin{matrix} -2,13 & 0 & 12.66 \\ 0.98 & 3.1 & -5.84 \\ 16.52a^{-1} & 0 & -50.64a^{-1} \end{matrix} \right\| \left\| \begin{matrix} V^{i,0}_1 \\ V^{i,0}_2 \\ V^{i,0}_3 \end{matrix} \right\| \tag{3.7}$$

The intermediate functions $V^{i,0}_1$, $V^{i,0}_2$, $V^{i,0}_3$ are expressed in terms of a matrix product

$$\| V^{i,0}_1, V^{i,0}_2, V^{i,0}_3 \|^T = V^0 \| M^x_{i,1}, K^x_{i,1}, Q^x_{i,1}, \beta^x_{i,1}, \gamma^x_{i,1}, W^x_{i,1} \|^T \tag{3.8}$$

$$V^0 = \left\| \begin{matrix} 0,15 & 0,325 & 0 & 0 & \xi & 0 \\ 0,5 & 0 & 0,125a & \xi & 0 & 0 \\ 0,163 & 0 & 0,021a & 0,5\xi & 0 & \xi a^{-1} \end{matrix} \right\|$$

The force factors at the right section of the element are determined from the equilibrium equations (2.1) together with expressions (3.4)

$$\left\| \begin{matrix} M^x_{i+1,j} \\ K^x_{i+1,j} \\ Q^x_{i+1,j} \\ N^x_{i+1,j} \end{matrix} \right\| = \left\| \begin{matrix} 0 & 1 & 2^{-1}a & 1 & 0 & a & 0 & 0 & -1 & -2^{-1}a & 2^{-1}a \\ 1 & 0 & 2^{-1}a & 0 & 1 & 0 & 0 & -1 & 0 & 2^{-1}a & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{matrix} \right\| \times$$

$$\times \| M^y_{i,j}, K^y_{i,j}, Q^y_{i,j}, M^x_{i,j}, K^x_{i,j}, Q^x_{i,j}, N^x_{i,j}, M^y_{i,j+1}, K^y_{i,j+1}, Q^y_{i,j+1}, F \|^T \tag{3.9}$$

Substituting the force factors (3.4), (3.9) into expressions (3.1) and similar expressions with cyclic permutation of the x and y indices we find the values of the deformations at the upper and right sections of the element (Fig. 2): $\beta^y_{i,j+1}$, $\gamma^y_{i,j+1}$, $W^y_{i,j+1}$, $\beta^x_{i+1,j}$, $\gamma^x_{i+1,j}$, $W^x_{i+1,j}$, $S^x_{i+1,j}$.

Thus the boundary conditions at the lower and left sections of the element are transferred to the upper and right sections of the element.

In going over to the next element of the panel, situated to the right of the element under consideration, to simplify the expressions the mechanical parameters have to be replaced by an equivalent system oriented along the axes of a new local system of coordinates $O'x'y'z'$ (Figs 1 and 2). Let $\alpha_{i,j}$ be the angles between the element planes, defined by the geometry of the panel. Then the forces and displacements with respect to the coordinates of the right-hand element, distinguished by primes, are expressed in the form

$$\left\| \begin{matrix} Q^{x'}_{i+1,j} \\ N^{x'}_{i+1,j} \\ W^{x'}_{i+1,j} \\ S^{x'}_{i+1,j} \end{matrix} \right\| = \left\| \begin{matrix} A_{i,j} & 0 \\ 0 & A^T_{i,j} \end{matrix} \right\| \left\| \begin{matrix} Q^x_{i+1,j} \\ N^x_{i+1,j} \\ W^x_{i+1,j} \\ S^x_{i+1,j} \end{matrix} \right\|, \quad A_{i,j} = \left\| \begin{matrix} \cos\alpha_{i,j} & \sin\alpha_{i,j} \\ -\sin\alpha_{i,j} & \cos\alpha_{i,j} \end{matrix} \right\| \tag{3.10}$$

The moments and angular deformations $M^x_{i+1,j}$, $K^x_{i+1,j}$, $\beta^x_{i+1,j}$, $\gamma^x_{i+1,j}$ remain unchanged in the new system $O'x'y'z'$.

Here the sweep over one element is now complete, and the above operations are performed for the next element along the vertical strip. After finishing this strip we pass to the next strip on the right, starting with the lowest element (Fig. 1). After reaching the upper boundary of the panel the parameters on it are entered into the vector of unknown parameters (3.2). In this case the action (3.4) is not performed.

We determine the solving system of linear algebraic equations in the mechanical sweep method by the boundary conditions at the upper and right edges of the rigidly clamped panel. These are $7n$ equations, which we represent by a column-matrix

$$\|\beta^y_{1,n+1}, \gamma^y_{1,n+1}, W^y_{1,n+1}, \ldots, \beta^y_{n,n+1}, \gamma^y_{n,n+1}, W^y_{n,n+1}, \beta^x_{n+1,1},$$
$$\gamma^x_{n+1,1}, W^x_{n+1,1}, S^x_{n+1,1}, \ldots, \beta^x_{n+1,n}, \gamma^x_{n+1,n}, W^x_{n+1,n}, S^x_{n+1,n}\|^T = 0 \tag{3.11}$$

$7n$ unknowns can occur in each equation, written in order in (3.2). The last component of the vector (3.2), depending on the force F, determines the free terms of the system of equations. Solving the system of equations (3.11), we obtain the values of the unknown parameters (3.2) corresponding to the specified load q. Then, repeating from the beginning the above-listed actions of the two-dimensional sweep, we obtain an array of data—the internal force factors and deformations of the panel. Here the parameters are specified without resolution into the components of (3.2).

In the present version of the sweep method the boundary conditions are transferred over a finite-element decomposition. One of the advantages which have led to the wide application of the finite-element method is the rapid convergence of errors to zero as the density of the decomposition grid is increased. Hence the proposed mechanical sweep method should be more accurate than previous sweep methods based on replacing derivatives in differential equations by finite differences, and which have the problem that the error increases as the grid step is reduced.

To calculate panel deformations depending non-linearly on the pressure P, we use the successive loading method. We determine the growth of internal and boundary force factors and deformations by making small increments in the panel pressure q. The total values of the internal factors and displacements, which are denoted by a bar in what follows, are given by algebraic summation of the increments in these parameters with respect to the step number of the loading

$$\overline{M}^x_{i,j} = \sum^k M^x_{i,j} \ldots \overline{W}^x_{i,j} = \sum^k W^x_{i,j} \ldots \tag{3.12}$$

The active load—the pressure P on the panel, is found from the conditions for static equilibrium of the panel. To do this along the line of largest displacements, in this case along the middle generator, we mentally cut the panel (Fig. 3). We introduce the l axis, parallel to the l_1 axis generating the cylinder and passing through the point of largest displacement of the panel—its centre. Then, from the equation of moments about the l axis, for example, for the left half of the panel, in which boundary force factors and pressure forces occur, we obtain

$$P = -\left(a^2 \sum_{j=1}^{n} \sum_{i=1}^{\frac{1}{2}n} H_{i,j}\right)^{-1} \left\{ \sum_{j=1}^{n} [\overline{M}^x_{1,j} + \overline{M}^x_{\frac{1}{2}n+1,j} + M_l(\overline{N}^x_{1,j}) + M_l\left(\overline{N}^x_{\frac{1}{2}n+1,j}\right) + \right.$$
$$\left. + M_l(\overline{Q}^x_{1,j})] + \sum_{i=1}^{\frac{1}{2}n} [K^y_{i,1} + K^y_{i,n+1} + M_l(\overline{Q}^y_{i,1}) + M_l(\overline{Q}^y_{i,n+1})] \right\} \tag{3.13}$$

where $H_{i,j}$ is the moment arm of F $(F = Pa^2)$ applied to the centre of the element numbered (i, j) about the l axis; $M_l(\overline{N}^x_{1,j})$, $M_l(\overline{N}^x_{1/2n+1,j})$, $M_l(\overline{Q}^x_{l,j})$, $M_l(\overline{Q}^y_{i,l})$, $M_l(\overline{Q}^y_{i,n+1})$ are the moments of the boundary forces over half the panel about the l axis.

There is then a new loading step q.

The non-linear dependence of the pressure P on the displacement of the panel centre W_c is basically determined by the moments of the longitudinal forces $\overline{N}^x_{1,j}$, $\overline{N}^x_{1/2n+1,j}$. Once the panel displacements reach its chord H (Fig. 1), the force increments $N^x_{1,j}$ reverse direction.

The site of the l axis is chosen to be the place of largest panel displacements. Equation (3.13) is set up for that direction of the l axis for which P takes its lowest value. Applying this value of the total pressure P to the entire panel, we iteratively improve the values of the deformation and load for the remaining sections of the panel.

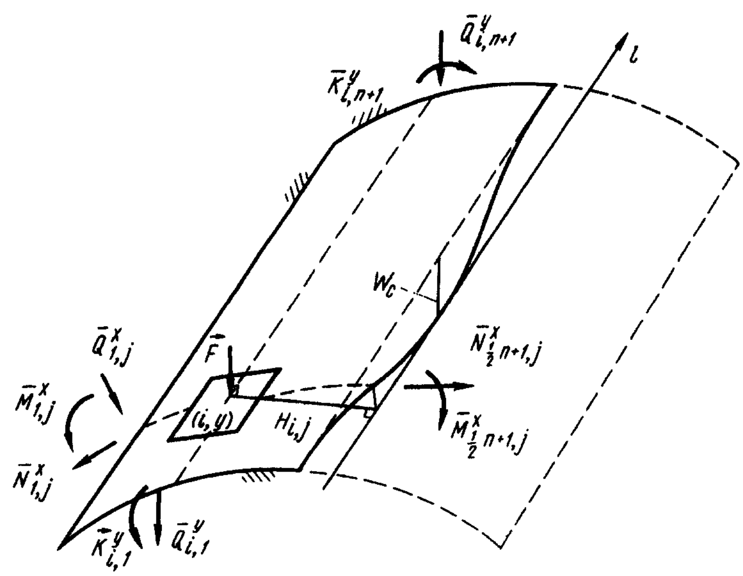

Fig. 3.

4. EXAMPLES

Calculations were performed for square titanium panels with the following data: $b = 0.1$ m; $R = 1$ m; $h = 8 \times 10^{-4}$ m; $E = 9.46 \times 10^{10}$ N m^{-2} (at a temperature of 200°C).

Figure 4 gives the calculated dependence (the solid curve) and the experimental dependence (the dashed curve) of the panel pressure on the central displacement W_c. Note their satisfactory agreement for initial non-linear deformations.

The accuracy of the calculation for large panel displacements can be preserved by reducing the number of degrees of freedom in the section element in fours and threes to the maximum number of six.

Figure 4(b) shows graphs of the distribution of the increase in longitudinal force $N^x_{1,j}$ per unit length along a generator coinciding with the edge of the panel at the first stage of the loading ($q = 4000$ N m^{-2}) (curve 1) and for displacements exceeding the chord of arc of the panel (curve 2); a graph of the increase in the shearing force $Q^y_{i,n+1}$ per unit length; and a graph of the panel displacements $W^x_{1/2n+1,j}$ along the median generators. It is clear that for the same increment of the pressure increases q the sign of $N^x_{1,j}$ changes when the panel displacements begin to exceed the chord of its arc.

Known analytic solutions for a rigidly clamped panel are associated with major simplifications and at the sub-critical deformation level give even smaller stiffness values than experiments [4].

Performing the mechanical sweep method for the case of a rigidly clamped square plate when there is a sufficiently accurate analytic solution showed differences of less than 3% when the plate was discretized into 25 elements. this also confirms the reliability of the method proposed.

5. MAIN RESULTS

We give a definition of the algorithm as a version of the sweep method for calculating mechanical systems.

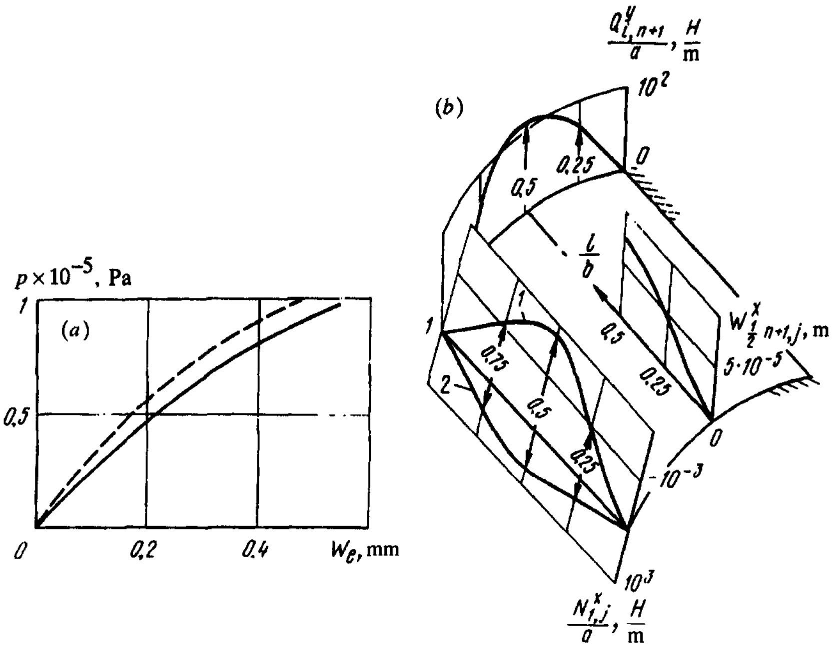

Fig. 4.

The mechanical sweep method consists in transferring boundary conditions, expressed in terms of mechanical parameters, over the elements into which the mechanical system is provisionally decomposed. Here, having first solved the equations for a restricted number of unknowns inside each element on then eliminates unknown parameters throughout the entire mechanical system.

When implementing this method on a computer the sweep parameters are conveniently specified in the form of three-dimensional arrays. The largest dimension of the array is equal to the dimensions for most parameters can be restricted to two. This enables one to separate out the construction onto a sufficiently large number of elements (~100) for working on microcomputers.

The use of an algorithm based on formula (3.13) has promise for the solution of geometrically non-linear mechanical problems. In this algorithm the active load is determined from the equilibrium condition of the separated deformed system when summing the internal force factors.

This method is primarily designed for calculating complex structures, such as bellows and aeroplane fuselage skins. It can also be generalized to three-dimensional problems.

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